Hopes and Beliefs in Financial Markets: Can Illusions Survive in the Long run?

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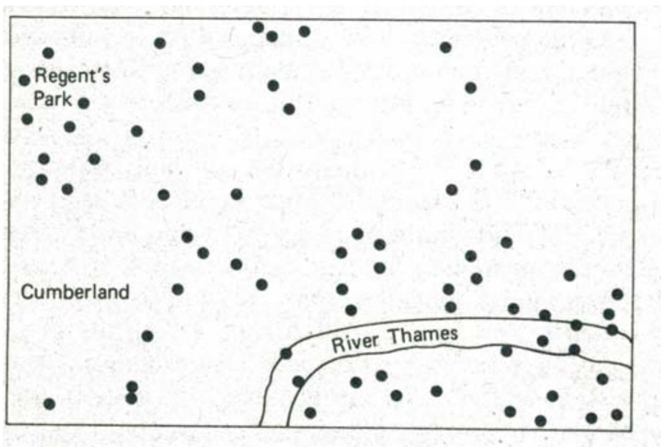
Abstract

This paper characterizes markets as an evolutionary environment where ambiguity and uncertainty are a common features. In that context, a new kind information traders named believers interact with traditional rational traders. This new type of trader affected by cognitive illusions misinterpret the relevance of their information in predicting future asset values. Using an evolutionary game theoretical set up, we show that the magnitude of the illusions affects traders' behavior and survival. Moreover when illusions are powerful, believers trade aggressively ensuring their survival in the long run.

Introduction

Based upon evidence from studies in psychology and decision making, several authors have investigated departures from full rationality in both economics and finance. Within this latter field, the common goal among these works is the attempt to explain anomalies in securities markets (see Barberis and Thaler [1] for a complete survey). Financial research has focused on several aspects of investor psychology resulting in incorrect expectations about asset payoffs. A great attention has been devoted so far to the role played by overconfidence in driving investors' misperception about returns. When predicting future prices, overconfident people set too narrow confidence bands and as a result get surprised more often than they anticipated. As a consequence, overconfident traders underestimate risk and trade more aggressively, as shown by De Long et al. [6] and more recently by Hirshleifer and Luo [10].

In this paper we focus on a different kind of bias, relying to the evidence that people often perceive relationships that in fact do not exist. Such behavior is known in the psychological literature as illusory correlation. Perhaps the London bombing during World War II constitutes the clearest illustration of such bias (see Gilovich [8] and the references therein). The points of impact of V-1 bombs appear to be randomly dispersed throughout London, as represented in Figure 1.



The impact of V-1 bombs in London during WWII (source: Gilovich [8])

Even so, Londoners asserted that some areas of the city were more dangerous than others because weapons hit the ground in clusters. This shows how people experiencing illusory correlation tend to find regularities in events that are truly random. Goldberg and Von Nitzsch [9] argue that illusions might explain initial public offerings in the high-tech sector. The first high-tech firms to be quoted in the NeuMarkt offered high returns due to their good future prospects. However investors incorrectly anticipated a positive relation between going public *per se* and high growth rate, rather than between the latter and sound future earnings.

As is clear illusions consist in establishing correlation among events which are unrelated. Consistently, we consider a model in which two classes of informed traders coexist. Rational traders constitute the first group, and correctly assess the relevance of their information in predicting asset returns. Believers experience illusory correlation, and constitute the second type of informed agents. These traders are boundedly rational because they misinterpret the relation between signals and the asset payoff. In analysing such trading enviromnent we closely follow the work by Hirshleifer and Luo [10], which consider similar interaction between rational and overconfident traders. In particular, we first focus on a static competitive setup and then introduce evolutionary dynamics to study the long-run properties of our static equilibrium.

Our main results relate both the trading activity and survival of believers to the illusion quality, which measure the degree of illusory correlation they experience. When the illusion quality is low, believers overestimate risk and as a consequence they take conservative positions in the risky asset. Such a behavior reduces their trading strategy profitability (with respect to rational traders), and believers do not survive in the long run. On the other hand, believers tend to trade more aggressively than rational agents when experiencing high quality illusions. This way believers are better equipped at exploiting profitable opportunities created by liquidity traders and the long-run equilibrium involves a positive fraction of believers. As is clear, introducing illusory correlation in financial markets proves to be flexible enough to encompass previous behavioral models such as Hirshleifer and Luo [10] on overconfidence, and Bernard and Thomas [2] on underconfidence.

The remainder of the paper is organized as follows. Section 1 defines the illusory correlation bias by reviewing the relevant contributions to the psychological literature. Section 2 characterize the static competitive equilibrium in a competitive financial market with rational traders and believers. The long-run properties of such a market are investigated in section 3. Finally section 4 concludes.

1 Illusory correlation

Illusory correlation is a cognitive illusion (or illusion of thinking) which shows a severe failure and inaccuracy in correlations assessments. This phenomenon was first documented by Chapman [3] and Chapman and Chapman [4], [5] in their work on word association and clinical psychologists (see also chapter 15 and 17 in Kahneman et al. [11]).

In their famous study Chapman and Chapman [5] showed their participants information concerning several hypothetical mental patients. The data for each patient consisted of a clinical diagnosis and a drawing of a person made by the patient. Later the participants estimate the frequency with which each diagnosis (such as paranoia or suspiciousness) had been accompanied by various features features of the drawings (such as peculiar eyes). Their finding was that the subjects markedly overestimate the frequency of co-occurrence of natural associates, such as suspiciousness and peculiar eyes. In their erroneous judgments of the data the participants "rediscovered" much of the common but unfounded clinical lore concerning the interpretation of the draw-a-person test. Moreover, the illusory correlation effect was so resistant to contradictory data that prevent them from detecting relationship that were in fact present.

As Chapman himself notice[3], illusory correlations are not restricted to the domain of clinical judgments. Most superstitions essentially are empirically groundless believes about the associations between particular actions or events and subsequent positive or negative outcomes. Racial, ethnic, regional, religious, or occupational stereotypes similarly are believes about covariations, beliefs that are strongly held and remarkably resistant to the impact of non-supporting data.

Chapman and Chapman studies were considered dramatic, controversial , and of considerable immediate relevance to the practitioner. Thus they originate a large body of studies concerned with the subjective correlation assessments that deviates more or less markedly from the correlation actually encountered.

The ability to figure out the correlations that hold between signals and their meanings, is a basic tool of adaptive intelligence. The psychology literature accounts for three classes of illusory correlations phenomena: (i) expectancy-based illusory correlations which suggest that observers tend to see the regularities that they do expect to find (ii) illusion arising from unequal weighting of information, which generally occurs when present events and committed behavior are deemed more important than absent events or omitted behaviors, and (iii) illusory correlations reflecting selective attention and encoding which happens when some observations catch more attention or are more likely to be encoded in memory and remembered than others. As is clear, all the three classes lead to the subjective overestimation of zero causal, complementary, or reciprocal relationship between two events. Consistently with the evidence contained in the psychological literature, we consider in the next section traders that misperceive the relevance of the information they possess in predicting future asset values.

2 Static model

2.1 Asset markets

Two securities are traded in a one-period competitive market: a riskfree asset with gross payoff normalized to 1, and a risky asset with final payoff f, where $f \sim N\left(\bar{f}, \sigma_f^2\right)$.

2.2 Agents

Three types of traders are active in the market: rational traders, believers, and noise traders. Rational traders and believers are informed traders, in that they receive some payoff relevant information (specified below, see section 2.3) before trading takes place. We normalize the size of informed traders to 1, and let λ and $1-\lambda$ denote respectively the fraction of rational traders and believers. In what follows the relevant variables for rational traders are denoted with subscript r (similarly for believers we use subscript b). Informed traders maximize expected utility over final wealth $\{w_i\}_{i=r,b}$. We assume that trader i's utility is exponential, $U(w_i) = -e^{-\gamma w_i}$, where $\gamma > 0$ denotes the absolute risk aversion coefficient, assumed equal across informed traders. Without loss of generality, informed traders are endowed with zero initial wealth, such that their final wealth is given by the gains from investing in the two assets. Since the risk-free security has a unity payoff, one has $w_i = x_i(f-p)$, where x_i is trader i's demand for the risky asset, and p is its price. Finally noise traders submit random demand $x \sim \left(0, \sigma_x^2\right)$ with x orthogonal to f.

2.3 Information structure

We denote each trader's information set by Ω_i, for $i = r, b$. Both rational traders and believers receive a noisy signal s of the final payoff, say $s = f + \varepsilon$, where $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$ and orthogonal to f. Rational traders correctly assess the signal's informational content, i.e. $\mathrm{Cov}_r(s, f) = \sigma_f^2$, as well as its precision, i.e. $\sigma_s^2 \equiv \mathrm{V}_r(s) = \sigma_f^2 + \sigma_\varepsilon^2$. On the other hand, believers misinterpret the relevance of s in providing payoff relevant information. More specifically, believers conjecture that $s = \alpha f + \eta$, where $\alpha > 0$ and $\eta \sim N\left(0, \sigma_\eta^2\right)$ is orthogonal to f. We refer to the parameter α as the illusion quality, since it measures the magnitude of the believers' misperception. In fact, the signal's informational content for a believer is given by $\mathrm{Cov}_b(s, f) = \alpha\sigma_f^2$. We assume that the signal's precision is correctly assessed by a believer, i.e. $\mathrm{V}_b(s) = \sigma_s^2$, and consequently set $\sigma_\eta^2 = \sigma_s^2 - \alpha^2\sigma_f^2$. For σ_η^2 to be well-defined, i.e. non-negative, we set $\alpha \in (0, \sigma_s/\sigma_f)$. As will be clarified in the section below, the assumptions on the uncertainty structure guarantee that when projecting f on the noisy signal, the regression coefficient used by a believer differs from the one estimated by a rational trader.

2.4 Equilibrium

Under our distributional assumptions on the final payoff and the signal, it follows that trader i's problem is given by:

$$\begin{array}{r} \max_{x_i} \ \mathrm{E}\left(w_i | \Omega_i\right) - \frac{\gamma}{2}\mathrm{V}\left(w_i | \Omega_i\right) \\ \text{s.t. } w_i = w + x_i\left(f - p\right). \end{array} \tag{1}$$

From the constraint in (1) it emerges that solving trader i's problem entails finding the first two moments of f conditional on the information set $\{\Omega_i\}_{i=r,b}$. Letting $\beta_r = \sigma_f^2/\sigma_s^2$ and making use of the Projection Theorem yields

$$\begin{array}{c} \mathrm{E}\left(f | \Omega_r\right) = \bar{f} + \beta_r\left(s - \bar{f}\right) \quad ; \quad \mathrm{E}\left(f | \Omega_b\right) = \bar{f} + \alpha\beta_r\left(s - \bar{f}\right) \quad ; \\ \mathrm{V}\left(f | \Omega_r\right) = \beta_r\sigma_\varepsilon^2 \quad \text{and} \quad \mathrm{V}\left(f | \Omega_b\right) = \beta_r\left(\sigma_s^2 - \alpha^2\sigma_f^2\right) \end{array} \tag{2}$$

such that trader i's demand function is given by

$$x_i = \frac{\mathrm{E}\left(f | \Omega_i\right) - p}{\gamma\mathrm{V}\left(f | \Omega_i\right)} \quad , \quad i = r, b. \tag{3}$$

The illusion quality α affects believers' trading behavior as follows. Suppose that $\alpha \in (1, \sigma_s/\sigma_f)$. From the conditional moments in (2) one has that $\beta_b = \alpha\beta_r > \beta_r$ and $\mathrm{V}(f | \Omega_b) <$

$\mathrm{V}(f|\,\Omega_r)$, such that eq. (3) gives $|x_b| > |x_r|$. When the illusion quality is large, believers overestimate the asset's conditional mean and underestimate its conditional variance. As a result, excessive volume would emerge due to believers taking larger positions than rational traders. The same behavior stems in Hirshleifer and Luo [10] due to overconfident traders. On the other hand, if $\alpha \in (0,1)$ then $\beta_b < \beta_r$ and $\mathrm{V}(f|\,\Omega_b) > \mathrm{V}(f|\,\Omega_r)$. In this case believers trade less aggressively than rational traders, i.e. $|x_b| < |x_r|$. Therefore a relatively low illusion quality is consistent with the trading behaviour of underconfident (or pessimist) traders.

Equipped with traders' demand functions, we now turn to derive the equilibrium price. Market clearing requires that

$$\lambda x_r + (1-\lambda)\, x_b + x = 0\ . \tag{4}$$

Substituting traders' demand (see eq. (3)) into the market clearing condition (4) gives the equilibrium price

$$p = \bar{f} + \psi^{-1}\varphi\beta_r\left(s - \bar{f}\right) + \psi^{-1}\gamma\beta_r\sigma_\varepsilon^2\left(\sigma_s^2 - \alpha^2\sigma_f^2\right)x \tag{5}$$

where φ and ψ are positive scalars given by

$$\varphi = \alpha\sigma_\varepsilon^2 + (1-\alpha)\,\lambda\left(\sigma_s^2 + \alpha\sigma_f^2\right)$$
$$\psi = \sigma_\varepsilon^2 + \left(1-\alpha^2\right)\lambda\sigma_f^2$$

Note from (5) that the equilibrium price is an unbiased estimate of the average final value $\bar{f}$, i.e. $E(p) = \bar{f}$. Moreover for $\lambda = 1$ the price is equivalent to the fully rational price p^r

$$p^r = \bar{f} + \beta_r\left(s - \bar{f}\right) + \gamma\sigma_\varepsilon^2\beta_r x$$

Equivalently p^r can be obtained setting $\alpha = 1$ in (5). In fact, both cases would correspond to all traders behaving rationally.

Trader i's unconditional profits are given by

$$E\left[\pi_i(\lambda)\right] = E(\pi_i) = \mathrm{E}\left(x_i(f-p)\right)\ .$$

Since the equilibrium price is centered around $\bar{f}$, profits coincide with the unconditional covariance between x_i and $(f-p)$.[1] Therefore:

$$E\left[\pi_r(\lambda)\right] = \frac{\sigma_\varepsilon^2}{\gamma\psi^2}\left[\left(1-\alpha^2\right)(1-\lambda)^2\sigma_f^2 + \gamma^2\beta_r\left(\sigma_s^2-\alpha^2\sigma_f^2\right)^2\sigma_x^2\right] \quad \text{and}$$

$$E\left[\pi_b(\lambda)\right] = \frac{\sigma_\varepsilon^2}{\gamma\psi^2}\left[-\left(1-\alpha^2\right)\lambda(1-\lambda)\sigma_f^2 + \gamma^2\beta_r\sigma_\varepsilon^2\left(\sigma_s^2-\alpha^2\sigma_f^2\right)\sigma_x^2\right] \ .$$

Using the above equations the difference in expected profits of the two types of traders $\Delta = \Delta(\lambda)$ is

$$\Delta(\lambda) = E\left[\pi_r(\lambda) - \pi_b(\lambda)\right] = \frac{\sigma_f^2\sigma_\varepsilon^2(1-\alpha)}{\gamma\psi^2}\left[(1-\alpha)(1-\lambda) + \gamma^2(1+\alpha)\beta_r\left(\sigma_s^2-\alpha^2\sigma_f^2\right)\sigma_x^2\right] \tag{6}$$

3 Model dynamics

We consider an evolutionary process for traders allowing types to replicate over time based on the profitability of their strategies. This criterion stems from the observation that traders use the strategies which turned out to be profitable in the past, thus imitating successful strategies. Suppose at time t the expected profit for a rational trader (resp. believer) is higher than the one for a believer (resp. rational trader); then at $t+1$ the proportion of rational traders increases (resp. decreases). If at time t both strategies yield the same expected profits, then the proportion of traders remain unchanged at $t+1$. The fraction of rational traders follows the dynamics

$$\lambda_{t+1} = \lambda_t + F\left(\Delta(\lambda_t);\lambda_t\right) \tag{7}$$

where the expected profit differential $\Delta(\lambda_t)$ is defined in (6). The function $F(\cdot):\mathbb{R}\times[0,1]\to[0,1]$ is assumed to be continuous and to satisfy the following properties (see Hirshleifer and Luo [10]):

i) $F(\cdot) = 0$ if $\Delta(\lambda_t) = 0$ and $\lambda_t \in (0,1)$

ii) $F(\cdot) < 0$ if $\Delta(\lambda_t) < 0$ and $\lambda_t > 0$

iii) $F(\cdot) > 0$ if $\Delta(\lambda_t) > 0$ and $\lambda_t < 1$

[1] Computing profits therefore boils down to taking covariances between the random variables f, s and x. As is clear, these covariances are the 'correct' ones, i.e. the ones a rational trader would correctly postulate.

iv) $F(\cdot) = 0$ if $\lim_{\lambda_t \to 0^+} \Delta(\lambda_t) \leq 0$

v) $F(\cdot) = 0$ if $\lim_{\lambda_t \to 1^-} \Delta(\lambda_t) \geq 0$

The equation for the fraction of rational traders in (7) together with the above properties essentially specifies a replicator dynamic model (see for example Fundenberg and Levine [7]). An alternative way to introduce dynamics in our model would be to keep the population fractions fixed and make the illusion quality α change over time. One could specify a dynamic equation for α similar to the one in(7) and embed it with properties equivalent to $i)-v)$ above. This alternative dynamics would be consistent with the idea that strategy profitability affect the bias, rather than population fraction. However there is abundant literature documenting that 'individuals are slow to change their beliefs in the face of new evidence' (see Shiller [13]) and are prone to ignore 'any information that conflicts with their point of view' (see Montier [12]). Both conservatism and confirmatory bias thus naturally lead our choice to dynamically model λ rather than α.

Conditions $i), ii)$ and $iii)$ describe the replicator's behavior when $\lambda_t \in (0,1)$, while the behavior at the extrema 0 (all believers) and 1 (all rational traders) is characterized by conditions *iv*) and *v*). We are interested in determining the existence and uniqueness of an equilibrium value for the fraction of informed traders, which we denote by λ^*. As is clear, this boils down to identify conditions under which the replicator equation (7) admits an interior fixed point viz. a corner solution.

We let the economy be represented by a vector of parameter values $\mathcal{E} = \left(\alpha, \gamma, \sigma_f^2, \sigma_\varepsilon^2, \sigma_x^2\right)$, and focus our attention on an admissible economies[2] as the ones characterized by $\mathcal{E} \in ((0, \sigma_s/\sigma_f) \setminus \{1\}) \times \mathbb{R}_+^4$. Like the following Proposition reveals, the illusion quality plays a crucial role in determining the equilibrium fraction of traders.

Proposition 1. For all admissible economies there exists a unique dynamic equilibrium given by:

1. $\lambda^* \in (0,1)$ if and only if $\alpha \in (\hat{\alpha}, \sigma_s/\sigma_f)$

2. $\lambda^* = 0$ if and only if $\alpha \in (1, \hat{\alpha})$

[2] We exclude $\alpha = 1$ as an admissible parameter value because in this case one cannot distinguish rational traders from believers.

3. $\lambda^* = 1$ if and only if $\alpha \in (0, 1)$

where $\hat{\alpha} \in (1, \sigma_s/\sigma_f)$ solves $\alpha^3 \kappa \beta_r + \alpha^2 \beta_r \kappa + \alpha (1 - \kappa) - (1 + \kappa) = 0$ and $\kappa = \gamma^2 \sigma_f^2 \sigma_x^2$.

According to Proposition 1, the survival of traders depends on the believers' misperception, as captured by α. Rational traders and believers coexist whenever the illusion quality is sufficiently high. The rationale behind the first finding is analogous to the one proposed by Hirshleifer and Luo [10]. In fact, high values of α imply that believers underestimate risk thus taking larger positions than rational traders. Thus believers better exploit the misvaluation that noise traders create in the market. Once the fraction of believers is large enough though, prices would move against them and rational traders would gain by trading in the opposite direction. As a result, both types of traders survive in the long run. Whenever the misperception is not very large (Proposition 1-part 2), believers still take more risky position than rational traders. However in this case believers engage in somewhat milder risk-taking trades. As a result prices do not move against them enough for rational traders to profitably take the opposite positions. Therefore believers are the only ones surviving in the long run. Finally, whenever believers underestimate the signal relevance in providing payoff-relevant information, i.e. $\alpha \in (0, 1)$, then they are driven out of the market (Proposition 1-part 3). Low values for α imply that believers overestimate risk, and as a result their trades are conservative. This way they are not able to exploit the misvaluation created by noise traders and thus do not achieve the returns enjoyed by rational traders. As a consequence they disappear in the long run.

Analysing how the fraction of believers surviving in the long run is affected by the underlying parameters is relevant. We have:

Corollary 1: Let $\alpha \in (\hat{\alpha}, \sigma_s/\sigma_f)$ such that $\lambda^* \in (0, 1)$ by Proposition 1-part 1; then the lower is the proportion of believers that survive in equilibrium

1. the lower is noise trading volatility (σ_x^2)
2. the higher is the illusion quality (α)

The comparative statics results stem from believers' risk underestimation whenever the illusion quality is sufficiently high, i.e. $\alpha \in (\hat{\alpha}, \sigma_s/\sigma_f)$. The source of believers' profits come from the misvaluation generated by noise traders. Believers are better equipped (with respect to rational traders) to profit from these opportunities because their trading is more aggressive. As a consequence, the fraction of believers is positively related to such profit opportunities as measured by the liquidity trading variance, σ_x^2. Similarly, we know that the illusion quality positively affects trading aggressiveness. When the illusion quality is very high, believers trade too aggressively and the fraction surviving in equilibrium decreases.

4 Conclusion

We propose a model where rational traders coexist with believers affected by illusory correlation. Such a psychological bias results in believers misinterpreting the relevance of their information in predicting future asset values. We show that the magnitude of illusory correlation affects traders' behavior and their ability to survive in the long run. When the illusion quality is low, believers trade too conservatively and do not survive in the long run. On the other hand, high illusion quality implies aggressive trading and enables a fraction of believers to survive in equilibrium.

Appendix

Proof (Proposition 1): An interior equilibrium value for the fraction of believers is defined by properties $i - iii$ of the $F(\cdot)$ function in the replicator (7). By property i the following must hold:

$$\lambda : \ \ \Delta(\lambda_t = \lambda) = 0$$

Using (6) one has

$$\lambda = 1 + \frac{(1+\alpha)\,\gamma^2 \beta_r \sigma_x^2 \left(\sigma_s^2 - \alpha^2 \sigma_f^2\right)}{1-\alpha} \tag{8}$$

As is clear, the numerator of the second term on the RHS in (8) is positive. Therefore $\lambda < 1$ if and only if $\alpha \in (1, \sigma_s/\sigma_f)$. For λ to be positive, the following condition must hold

$$g\,(\alpha; \beta_r, \kappa) = \alpha^3 \kappa \beta_r + \alpha^2 \beta_r \kappa + \alpha\,(1-\kappa) - (1+\kappa) > 0$$

where $\kappa = \gamma^2 \sigma_f^2 \sigma_x^2 > 0$. Notice that: 1) the terms in α^3 and α^2 are positive, 2) the constant is negative while 3) the sign of the linear term depends on $1-\kappa$. However, regardless of whether $\kappa > 1$ or $\kappa < 1$ there is always a single sign change in the coefficients of the polynomial $g\,(\alpha; \beta_r, \kappa)$. It follows by Descartes' rule that there is a unique positive root $\hat{\alpha}$ which solves $g\,(\hat{\alpha}; \beta_r, \kappa) = 0$. Moreover consider

$$\begin{aligned} g\,(\alpha = 1; \beta_r, \kappa) &= -2\sigma_\varepsilon^2 \\ g\,(\alpha = \sigma_s/\sigma_f; \beta_r, \kappa) &= \frac{\sigma_s}{\kappa \beta_r}\,(\sigma_s - \sigma_f) \end{aligned}$$

Therefore $g\,(\alpha = 1; \beta_r, \kappa) < 0 = g\,(\hat{\alpha}; \beta_r, \kappa) < g\,(\alpha = \sigma_s/\sigma_f; \beta_r, \kappa)$, yielding $\hat{\alpha} \in (1, \sigma_s/\sigma_f)$ and $\lambda \in (0,1)$ if and only if $\alpha \in (\hat{\alpha}, \sigma_s/\sigma_f)$. Finally it is straightforward to check that $\Delta\,(\lambda_t) > 0$ if and only if $\lambda_t < \lambda$ and $\Delta\,(\lambda_t) < 0$ if and only if $\lambda_t > \lambda$ (properties ii and iii). It follows that λ in (8) is indeed an interior fixed point for the dynamics defined in the main text.

We now consider corner solutions for the population dynamics (properties iv and v). The behavior of $\Delta\,(\cdot)$ at the boundaries is described by the following:

$$\lim_{\lambda_t \to 0^+} \Delta\,(\lambda_t) = \frac{\sigma_f^2 \sigma_\varepsilon^2}{\gamma \psi^2} \left[(1-\alpha)^2 + \gamma^2 \left(1-\alpha^2\right) \beta_r \left(\sigma_s^2 - \alpha^2 \sigma_f^2\right) \sigma_x^2\right] \tag{9a}$$

$$\lim_{\lambda_t \to 1^-} \Delta\,(\lambda_t) = \frac{\left(1-\alpha^2\right) \gamma \beta_r \sigma_f^2 \sigma_\varepsilon^2 \left(\sigma_s^2 - \alpha^2 \sigma_f^2\right) \sigma_x^2}{\psi^2} \tag{9b}$$

From (9a) it follows that if $\alpha \in (0, 1)$ then $\lim_{\lambda_t \to 0^+} \Delta(\lambda_t) > 0$. Suppose then $\alpha > 1$. The condition $\lim_{\lambda_t \to 0^+} \Delta(\lambda_t) \leq 0$ is equivalent to $g(\alpha; \beta_r, \kappa) \leq 0$. Therefore $\lambda^* = 0$ if and only if $\alpha \in (1, \hat{\alpha})$. Finally $\lim_{\lambda_t \to 1^-} \Delta(\lambda_t) \geq 0$ if and only if $\alpha \in (0, 1)$. ■

Proof (Corollary 1). At the interior fixed point λ^* :

$$\frac{\partial \lambda^*}{\partial \sigma_x^2} = \frac{(1+\alpha)\gamma^2 \beta_r \left(\sigma_s^2 - \alpha^2 \sigma_f^2\right)}{1-\alpha}$$

which is clearly negative, since $\alpha > \hat{\alpha} > 1$ at the interior fixed point. Taking the derivative of λ^* with respect to α gives

$$\frac{\partial \lambda^*}{\partial \alpha} = \frac{2\gamma^2 \beta_r \sigma_x^2}{(1-\alpha)^2} h(\alpha)$$

where $h(\alpha) = \sigma_f^2 (1+\alpha)(1-\alpha)^2 + \sigma_\varepsilon^2 > 0$, thus yielding $\frac{\partial \lambda^*}{\partial \alpha} > 0$. ■

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